

1 Three balls

States that the CM moves with a constant velocity $v/3$ Correctly writes the law of conservation of momentum	0.5 pts
Calculates v_A , v_B and v_C in the CM reference frame Correctly writes down kinematic relations for the two rods	0.9 pts
Calculates the kinetic energy $E = mv(2/3)$ Correctly writes the law of conservation of energy	0.6 pts
Calculates the angular momentum $L = mv\ell$ Correctly writes the law of conservation of angular momentum.	1.5 pts
Proves the impossibility of the case $\varphi = 0$	1 pt
States that the minimum distance is met when $d\varphi/dt = 0$ Comes to the conclusion that v_A and v_C are perpendicular to AC	0.5 pts
States that at this moment the rods AB and CB have identical angular velocities.	1 pt
Uses the relation $I = L^2/(2E)$ Uses $L = I\omega$ and $E = I\omega^2/2$	1 pt
States that the CM is in the medicenter	0.5 pts
Calculates I as a function of d or φ	1 pt
Equalizes the two expressions for I	0.5 pts
Finds the minimum value of d	1 pt

2 Solenoid

Stating that boiling starts when pressure becomes equal to p_s , the saturation pressure (full marks if used correctly implicitly)	1 pt
Neglecting p_s as compared to p_0 (full marks if used correctly implicitly)	0.5 pts
Neglecting water column pressure as compared to p_0 (full marks if used correctly implicitly)	0.5 pts
Concluding that drop due to magnetic forces must be equal to p_0 (full marks if used correctly implicitly)	1 pt
Showing that $p_0 = p + (\mu_r^{-1} - 1)B^2/(2\mu_0)$	4.5 pt
Partial score for failed attempt: using formula for magnetic field energy density $w = B^2/(2\mu_r\mu_0)$	1 pt
interaction energy $\Delta w = (\mu_r^{-1} - 1)B^2/(2\mu_0)$	1.5 pts
relating interaction energy difference to pressure difference	2 pts
Alternative approach with dipole-field interaction analysis	
Energy of a magnetic dipole \vec{d}_m in magnetic field \vec{B} : $-\vec{d}_m \cdot \vec{B}$	0.5 pts
Hence, force acting on a magnetic dipole (parallel to \hat{x}) in magnetic field (parallel to \hat{x}): $F = d_m \frac{dB}{dx}$	0.5 pts
Induced magnetic dipole moment density: $J = B\chi/(\mu_r\mu_0)$	0.5 pts
Hence, magnetic force per volume $f_m = B\chi(\mu_r\mu_0)^{-1} \frac{dB}{dx}$	0.5 pts
This can be rewritten as $f_m = \frac{1}{2}\chi(\mu_r\mu_0)^{-1} \frac{dB^2}{dx}$	0.5 pts
Magnetic force is balanced with the pressure force per volume $f_p = -\frac{dp}{dx}$	0.5 pts
Hence $\frac{d}{dx} [-p + \frac{1}{2}\chi(\mu_r\mu_0)^{-1}B^2] = \text{const}$	0.5 pts
Hence $p_0 - p = -\frac{1}{2}\chi(\mu_r\mu_0)^{-1}B^2$	0.5 pts
Remark: if the pressure is calculated as a pressure from induced solenoidal currents (due to water magnetization) near the side walls of test tube, only 2 point out of 4.5 is given (because the pressure at the water-wall interface is unknown).	
Using or deriving formula for the magnetic field inside a long solenoid $B = IN\mu_0/\ell$	1 pts
Using the above results, expressing I	1 pts
Remark: this point can be given only if the solution is correct, except for a possible mistake by a factor of $\sqrt{2}$	
Evaluating I numerically	0.5 pts
Remark: this point can be given only if the solution is correct, except for a possible mistake by a factor of $\sqrt{2}$	

3 Staircase

A (2 points in total for part A)

1. $x(n) = n^{2/3}\lambda$ (1 pt).
2. $d_n = x(n+1) - x(n) = \lambda[(n+1)^{2/3} - n^{2/3}]$ (0.5 pt)
For $n \gg 1$, $d_n = (2/3)\lambda n^{-1/3}$ (0.5 pt)

Other solutions leading to the correct answer without computing x_n are accepted. 0.5 pt for final expression of d_n are awarded only if both the prefactor and the exponent are correct.

B (8 points in total for part B)

1. Minimal energy principle expressed mathematically (1 pt)
2. Idea of minimization against small changes in shape (1 pt)
3. Volume conservation principle (2 pt).
4. Energy cost for displacements of one step, $\epsilon_n(\delta)$ or equivalent, computed correctly (2 pt).
5. Combination of energy minimization and volume conservation in mathematically correct form (1 pt).
6. Correctly derived final answer $\nu = -2$ (1 pt).