

## Zero-length springs and slinky coils – Marking Scheme

## Part A: Statics

A.1	First method:	
	realized that $\frac{\Delta y}{\Delta l} = \frac{L}{L_0}$	0.2 pts
	realized that $L = \frac{F}{k}$	0.1 pts
	mentioned that $\Delta y = \Delta l$ if $F < kL_0$	0.1 pts
	correct result	0.1 pts
	Second method:	
	realized $k^* = k \frac{L_0}{\Delta l}$	0.2 pts
	mentioned that $\Delta y = \Delta l$ if $F < kL_0$	0.1 pts
	used $F - F_0 = k^* (\Delta y - \Delta l)$	0.1 pts
	correct result	0.1 pts
A.2	used $F(x) = \frac{kL_0}{\Delta l} x$	0.1 pts
	used $\Delta W = \int_{\Delta l}^{\Delta l_2} \frac{kL_0}{\Delta l} x dx$ (if $\Delta W = \frac{kL_0}{\Delta l} (\Delta y - \Delta l)^2$ – no pts.)	0.3 pts
	correct result	0.1 pts
A.3	calculating $l_0$ , the length of the closed turns	0.2 pts
	correct result for $l_0$	0.3 pts
	realized $F(l) = Mg \frac{l}{L_0}$	0.3 pts



	expressed $dy = \frac{Mg}{kL_0^2} l dl$	0.2 pts
	used $H = l_0 + \int_{l_0}^{L_0} \frac{Mg}{kL_0^2} l dl$ (if assumed $l_0 = 0$ : 0.1 pts)	0.3 pts
	correct result expressed using the required variables (if assumed $l_0 = 0$ : 0.4 pts)  correct result but not expressed using the required variables (if assumed $l_0 = 0$ : 0.1 pts)	0.7 pts  0.4 pts

**Part B: Dynamics**

B.1	used $H(l) = \frac{l^2}{2l_0} + \frac{l_0}{2}$	0.3 pts
	the contribution of element $dl$ to the center of mass: $\left( \frac{l^2}{2l_0} + \frac{l_0}{2} \right) \frac{Mdl}{L_0}$	0.2 pts
	$H_{\text{cm}} = \frac{1}{M} \left[ \frac{l_0}{2} \alpha M + \int_{l_0}^{L_0} H(l) dm \right]$ (first term 0.3, second term 0.3)	0.6 pts
	correct calculation of $H_{\text{cm}} = L_0 \left[ \frac{1}{6\alpha} - \frac{\alpha^2}{6} + \frac{\alpha}{2} \right]$ (if assumed $l_0 = 0$ : 0.3 pts)	0.4 pts
	found the displacement of the center of mass: $\Delta H = H_{\text{cm}} - \frac{L_0}{2}$	0.2 pts

	used $\frac{g}{2}t_c^2 = H_{cm} - \frac{L_0}{2}$	0.2 pts
	correct result expressed using the required variables	0.5 pts
	correct result but not expressed using the required variables	0.4 pts
	numerical value of time	0.1 pts
B.2	Default method – work of external force equals the change in kinetic energy	
	calculated the position of the upper part ( $l < l' < L_0$ ) before the spring is released, used $H(l') = \frac{l'^2}{2l_0} + \frac{l_0}{2}$	0.2 pts
	$H_{cm-upper} = \frac{L_0}{M(L_0 - l)} \int_l^{L_0} H(l') dm = \frac{L_0}{M(L_0 - l)} \int_l^{L_0} \left( \frac{l'^2}{2l_0} + \frac{l_0}{2} \right) \frac{M dl'}{L_0}$ if using $M$ instead of $\frac{M(L_0-l)}{L_0}$ : only 0.1 pts	0.3 pts
	found $H_{cm-upper} = \frac{L_0^2 + L_0 l + l^2}{6l_0} + \frac{l_0}{2}$	0.5 pts
	found the position of the center of mass of the moving part: $H_{cm-upper-f} = \frac{l^2}{2l_0} + \frac{l_0}{2} + \frac{1}{2}(L_0 - l)$ (first two terms – 0.1 pts, third term – 0.2 pts)	0.3 pts
	$\Delta H_{cm-upper} = H_{cm-upper} - H_{cm-upper-f} = \frac{(L_0 - l)(L_0 + 2l)}{6l_0} - \frac{1}{2}(L_0 - l)$ (finding the difference – 0.1 pts, result for the difference – 0.2 pts)	0.3 pts
	understood that the external force is $Mg$ (any other expression – 0 pts)	0.3 pts
	Using $Mg\Delta H_{cm-upper} = \frac{M(L_0-l)}{2L_0} v^2$	0.3 pts



	obtained $v_{upper-f}^2 = \frac{2g}{3\alpha}l + \frac{L_0g}{3\alpha} - L_0g$	0.3 pts
	Other methods: There are other ways to solve this part Finding A and B without deriving $v_1(l) = \sqrt{Al + B}$	1.3 pts
B.3	realized that the velocity decreases in time when a stationary part still exists  in case a student gets $A < 0$ : 0.1 pts for the opposite conclusion	0.1 pts
	substituted $l = l_0$ to find $v_1(l_0) = \sqrt{Al_0 + B}$	0.1 pts
	used conservation of momentum (plastic collision) to find correct result  $v_{min} = \frac{m_{top}v(l_0)}{M}$	0.3 pts

## Part C: Energetics

C.1	an equation relating $Q$ and $W$ (without gravitational energy)	0.4 pts
	use $dW = \frac{kL_0}{2dl} \left( \left( \frac{1}{l_0} l dl \right)^2 - dl^2 \right) = \frac{kL_0}{2} \left( \frac{l^2}{l_0^2} - 1 \right) dl$	0.4 pts
	calculated the elastic energy of the spring: $W = \int_{l_0}^{L_0} \frac{kL_0}{2} \left( \frac{l^2}{l_0^2} - 1 \right) dl$	0.4 pts
	correct result $W = MgL_0 \frac{(1-\alpha)^2(2\alpha+1)}{6\alpha}$	0.3 pts