Thermoacoustic engine – Marking Scheme

Part A: Sound wave in a closed tube

A.1	boundary conditions $u(L,t) = 0$	0.2pts
	$\lambda_{max} = 2L$	0.1 pts
A.2	$V(x,t) = S \cdot \Delta x_t(t)$	0.1 pts
	comparing two sides $\frac{1}{\Delta x}\big(u(x+\Delta x,t) - u(x,t)\big) = \frac{du}{dx}$	0.2 pts
	final result	0.2 pts
A.3	$\Delta F = S \cdot \Delta p$	0.1 pts
	$\Delta p \approx -\frac{dp(x,t)}{dx} \cdot \Delta x$	0.2 pts
	Newton's 2nd law $\Delta F = m\frac{d^2u}{dt^2} \approx \rho_0 \cdot S \cdot \Delta x \frac{d^2u}{dt^2}$	0.2 pts
	final result	0.2 pts
A.4	final result	0.3 pts
A.5	ideal gas $pV = nRT$	0.2 pts
	$T = \frac{T_0}{p_0V_0}\big(p_0 - p_1\,cos\,(\omega t)\big)\big(V_0 + V_1\,cos\,(\omega t)\big)$	0.1 pts
	$T = T_0 + T_0(V_1/V_0 - p_1/p_0)\,cos\,(\omega t)$	0.2 pts
	final result	0.2 pts
A.6	Two correct sites	0.6 pts
	All are correct	1.2 pts

Part B: Sound wave amplification induced by external thermal contact

B.1	$T_{env}(t) = T_0 - \frac{\tau}{l} \cdot \big(u(L/4,t)\big)$	0.3 pts
	final result	0.1 pts
B.2	$T_{st} > T_1$	0.6 pts
	final result	0.4 pts
B.3	use first law $\Delta Q = \Delta E + W$ or $\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$	0.4 pts

	understand energy of ideal gas $\Delta E = c_v n \Delta T$	0.1 pts
	$nR\Delta T = p\Delta V + \Delta p \cdot V$	0.2 pts
	final result	0.1 pts
B.4	final result $V_b = \frac{1}{\gamma} p_b \frac{V_0}{p_0}$	0.5 pts
	$V_a = \left[\frac{\gamma-1}{\gamma}\frac{\beta}{\omega}(T_{st} - T_1) - \frac{1}{\gamma} p_a\right]\frac{V_0}{p_0}$	0.8 pts
	$T_{st} - T_1 = (\tau - \tau_{cr})\frac{a}{l\sqrt{2}}$	0.5 pts
	final result $V_a = \left[-\frac{\gamma-1}{\gamma}\frac{\beta}{\omega}(\tau - \tau_{cr})\frac{a}{l\sqrt{2}} - \frac{1}{\gamma} p_a\right]\frac{V_0}{p_0}$	0.1 pts
B.5	$w = \int p dV = \int_0^{\Delta t_c} p \frac{dV}{dt} dt$	0.2 pts
	$\int_0^{\Delta t_c} cos\,(\omega t) dt = \int_0^{\Delta t_c} sin\,(\omega t) dt = \int_0^{\Delta t_c} sin\,(\omega t)$ $cos\,(\omega t) dt = 0$ (or equivalent)	0.2 pts
	$\int_0^{\Delta t_c} cos^2\,(\omega t) dt = \int_0^{\Delta t_c} sin^2\,(\omega t) dt = \frac{\Delta t_c}{2}$	0.2 pts
	$w = -\Delta t_c \frac{\omega}{2}(p_a V_b + p_b V_a) = -\pi(p_a V_b + p_b V_a)$	0.1 pts
	$W = \frac{\pi}{2\omega}(\gamma - 1)\beta(\tau - \tau_{cr}) k a^2 S$	0.1 pts
B.6	$Q_{tot} = S\int_0^{\Delta t_c} j \cdot dt = \frac{1}{\Delta x}\int_0^{\Delta t_c} Q \frac{du}{dt} \cdot dt$	0.2 pts
	$Q = -\frac{1}{\omega}\beta V_0 (T_{st} - T_1)\,sin\,(\omega t) + Q_0$	0.1 pts
	$\frac{du}{dt}(x,t) = -a\omega \cdot sin\ (kx)\,sin\,(\omega t)$	0.1 pts
	$Q_{tot} = \frac{1}{\omega}\pi a \beta S (T_{st} - T_1)\,sin\,(kx)$	0.3 pts
	$Q_{tot} = \frac{1}{\omega}\frac{\pi}{2} a^2 \beta \frac{S}{l}(\tau - \tau_{cr})$	0.1 pts
B.7	$\eta = (\gamma - 1)kl$	0.2 pts
	$\eta = \frac{\tau_{cr}}{T_0} = \frac{\tau_{cr}}{\tau}\frac{\tau}{T_0} \approx \eta_c \frac{\tau_{cr}}{\tau}$	0.4 pts

Note that some students may solve B.6 and B.7 using a different method. A student that solved correctly B.7, and inferred incorrect answer for B.6 with correct units, will get full points for B.7 and ½ of the points for B.6.