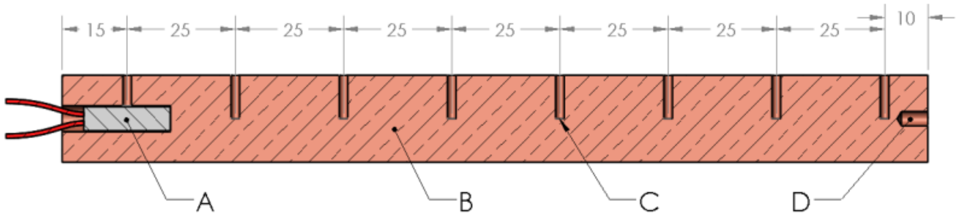
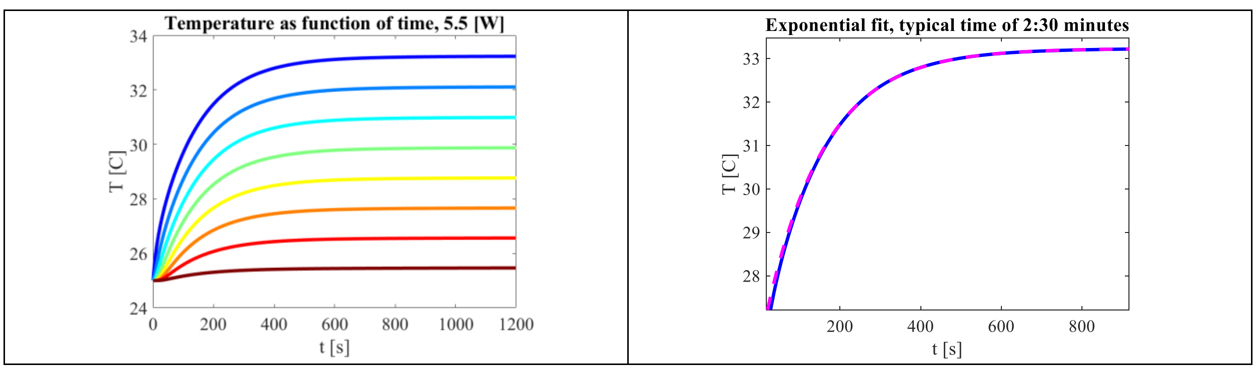
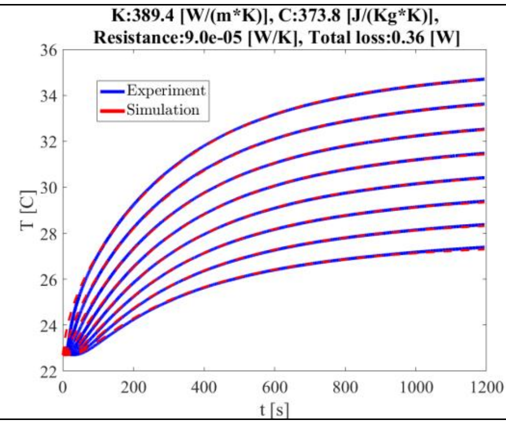
Wiedemann-Franz law – Appendix



We will use a simple model to analyze the heat transport through the rod. A heater is suppling power P_0 at one end. At any given point $P = -\kappa A \frac{\partial T}{\partial x}$ where P is the heat current at point x. Ideally, PT should be constant along the rod. However, there are two mechanism that modify this: (a) loss of power that is used to change the temperature of the material; (b) loss of power to the environment, leading to $\frac{\partial P}{\partial x} = -c_p \cdot \rho \cdot A \frac{\partial T}{\partial t} - (T - T_0) \cdot K$, where c_p is the specific heat, ρ the density, K the radial heat loss parameter per unit length, and T_0 the environment temperature.



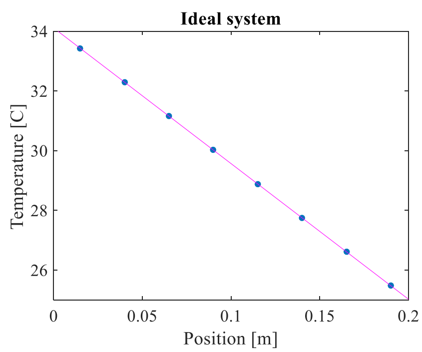
The time scale of the heating process is 2:30 minutes, so we get to 95% of final value after 7:30 minutes, as shown in the figure above. However, this is again only true for an ideal system properly coupled to a heat reservoir but otherwise insulated. In our system, the connection to the heat reservoir is not ideal, and we have radial heat loss. We model this system as a rod connected via thermal resistance to a short aluminum rod (representing the pot and heat sink material in the real setup) that is connected to a water reservoir. In the figure below we have simulated the realistic heating process, and compared it with the measured data.



During the experiment we will see the numbers that are extracted from this simulation match our experimental results.

Part A

For an ideal system, a rod connected only to an ideal reservoir, we expect to get a constant temperature gradient:

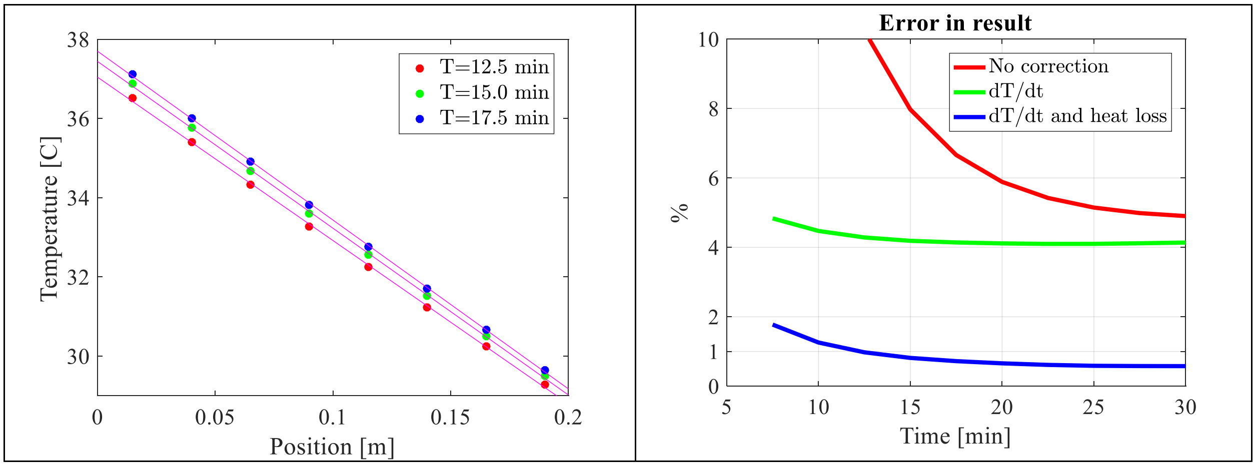


Which gives $\kappa = \dfrac{P}{Slope \cdot A} = 385\dfrac{W}{m \cdot K}$

Let us now analyze the effects of the two mechanisms causing deviation from this ideal behavior. An important point here is that loss mechanisms affect the heat flux, but there is always a local relation between the temperature gradient and the local heat. Hence, the temperature gradient close to the heater is set by input power P, while the gradient close to the reservoir side will be set by power $P - P_{loss}$. For this reason, the slope will be corrected by (to first order in the gradient):

$$\kappa = \frac{P - \frac{1}{2}P_{absorb} - \frac{1}{2}P_{loss}}{\vec{\nabla}T \cdot A} = \frac{P - \frac{1}{2}c_p \cdot m \cdot \frac{\partial T}{\partial t} - \frac{1}{2}P_{loss}}{\vec{\nabla}T \cdot A}$$

The figure below demonstrates that this approximation is very accurate for our system:

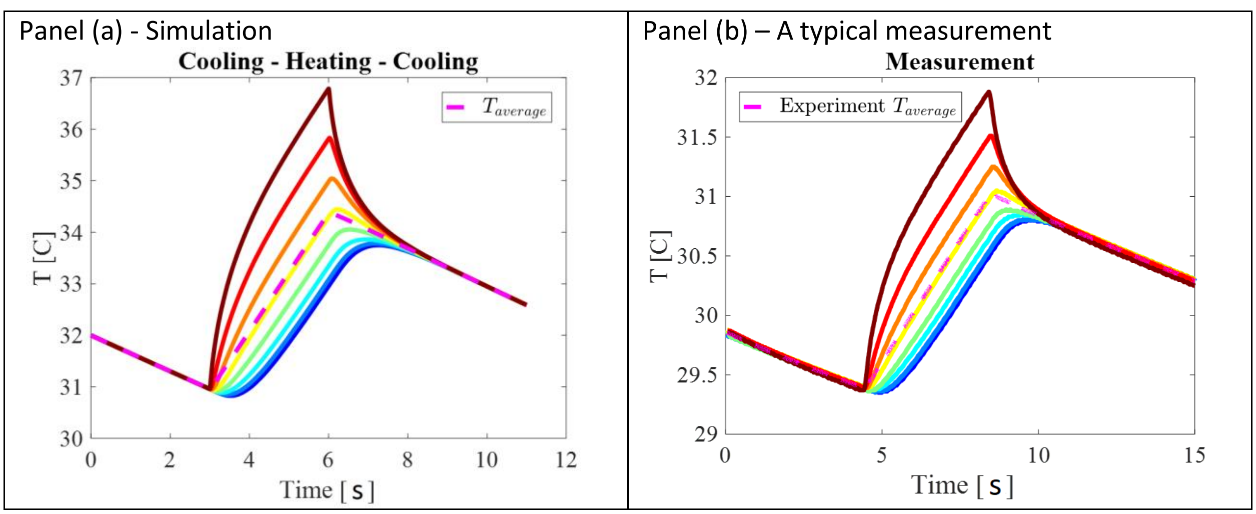


We see that with these two correction the accuracy is better than 1%, compared with 8% without correction, which is an order of magnitude improvement.

Part B: cooling – heating – cooling cycle

The purpose of this section is to obtain the heat loss and heat capacity of the rod in order to correct the 8% error in the previous section result. To do so, we will disconnect the rod from the heat reservoir, isolate its free end, and heat it up to approximately the temperature of the last section. In this section we do not aim for a 1% accuracy, because it is a first order correction for our result of the heat conductance, which is the main point of the experiment.

We will perform a cooling-heating-cooling cycle in order to obtain the heat capacity and the heat loss.



Panel (a) – a simulation of the cooling-heating-cooling cycle. Panel (b) – a typical cycle as measured in a real system. The different colors correspond to different thermometers.

The average reading (over all thermometers) is in principle the most suitable quantity for the analysis, since it approximately accounts for the total energy given to the rod by the heater. Unfortunately, we believe that it is technically difficult to write the full set of thermometer readings fast enough on paper, and we do not want the student to invest a lot of effort in this task. To a very good approximation the temperature in the center of the rod is very close to the average temperature, up to some small time delay. Hence, we hint that the student should use the temperature in the center (average of readings of thermometers T4 and T5).

There are a number of ways to use this data in order to extract the two quantities C, P_{loss}. We propose two approaches: (a) based on the slope of the $T(t)$ measurements during the cooling and heating process. In the former,

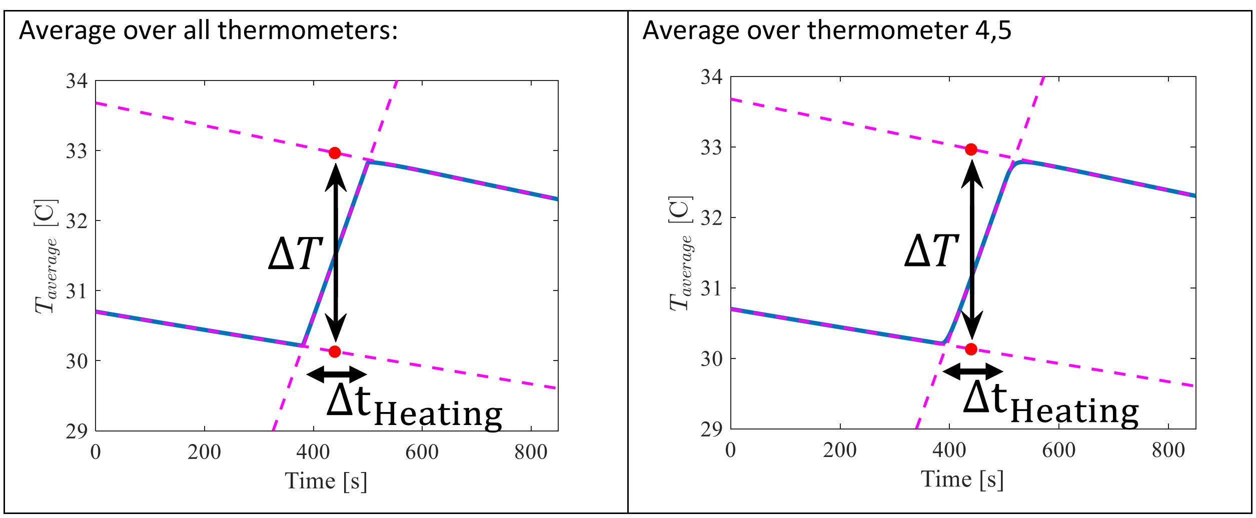
$$P_{loss} = -c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right|_{Cooling}$$

Hence in the latter:

$$P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right|_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right|_{Cooling} \right),$$

allowing one to extract $c_p \cdot m$.

(b) Alternatively, the total heat input during the heating time interval, $P_{\text{in}} \cdot \Delta t_{\text{Heating}}$ is related to the $\Delta T_{\text{Heating}}$, the difference between the vertical shift between the linear cooling $T(t)$ graphs before and after the heating (see figure below) through $P_{\text{in}} \Delta t_{\text{Heating}} = c_p m \Delta T_{\text{Heating}}$. Since the slope during the relatively short heating time interval is not needed here, this method is slightly more accurate. However, both approaches will be accepted.



Both methods reproduce the heat conductivity within 1%, $386\left[\frac{J}{kg\cdot K}\right]$.

Using this heat capacity we can get the total heat loss $P_{loss} = -c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right|_{Cooling} = 0.34\left[\frac{J}{s}\right]$

Note that a significant contribution to the heat capacity originates from the insulation and other components in the system, and for this reason we instruct the students to use an effective mass that is larger than the real mass. This matters only in order to get proper c_p , but it is irrelevant for the rest of the experiment where we only need $c_p \cdot m$.

Depending on the method, the result of C will deviate by 5%. This will produce less than $\sim 0.5\%$ negligible error in the result of the heat conductivity.