Problem 3 : Solution/marking scheme – Large Hadron Collider *(10 points)*

Part A. LHC Accelerator (6 points)

A1 (0.7 pt) Find the exact expression for the final velocity v of the protons as a function of the accelerating voltage V, and fundamental constants.

Solution A1: **[0.7]**

Conservation of energy:

$$m_p \cdot c^2 + V \cdot e = m_p \cdot c^2 \cdot \gamma = \frac{m_p \cdot c^2}{\sqrt{1 - v^2/c^2}}$$ **0.5**

Penalties

No or incorrect total energy **-0.3**

Missing rest mass **-0.2**

Solve for velocity:

$$v = c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e}\right)^2}$$ **0.2**

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without proton rest mass: **[0.5]**

$$V \cdot e \simeq m_p \cdot c^2 \cdot \gamma = \frac{m_p \cdot c^2}{\sqrt{1 - v^2/c^2}}$$ **0.3**

Solve for velocity:

$$v = c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{V \cdot e}\right)^2}$$ **0.2**

Classical solution: **[0.2]**

$$v = \sqrt{\frac{2 \cdot e \cdot V}{m_p}}$$ **0.2**

A2 (0.8 pt) For particles with high energy and low rest mass the relative deviation $\Delta = (c - v)/c$ of the final velocity v from the speed of light is very small. Find a suitable approximation for Δ and calculate Δ for electrons with an energy of 60.0 GeV.

Solution A2: **[0.8]**

velocity (from previous question):

$$v = c \cdot \sqrt{1 - \left(\frac{m_e \cdot c^2}{m_e \cdot c^2 + V \cdot e}\right)^2} \quad \text{or} \quad c \cdot \sqrt{1 - \left(\frac{m_e \cdot c^2}{V \cdot e}\right)^2} \qquad \mathbf{0.1}$$

relative difference:

$$\Delta = \frac{c - v}{c} = 1 - \frac{v}{c} \qquad \mathbf{0.1}$$

$$\rightarrow \Delta \simeq \frac{1}{2}\left(\frac{m_e \cdot c^2}{m_e \cdot c^2 + V \cdot e}\right)^2 \quad \text{or} \quad \frac{1}{2}\left(\frac{m_e \cdot c^2}{V \cdot e}\right)^2 \qquad \mathbf{0.4}$$

relative difference

$$\Delta = 3.63 \cdot 10^{-11} \qquad \mathbf{0.2}$$

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classical solution gives no points **0.0**

A3 (1.0 pt) Derive an expression for the uniform magnetic flux density B necessary to keep the proton beam on a circular track. The expression should only contain the energy of the protons E, the circumference L, fundamental constants and numbers. You may use suitable approximations if their effect is smaller than the precision given by the least number of significant digits. Calculate the magnetic flux density B for a proton energy of $E = 7.00$ TeV.

Solution A3: **[1.0]**

Balance of forces:

$$\frac{\gamma \cdot m_p \cdot v^2}{r} = \frac{m_p \cdot v^2}{r \cdot \sqrt{1 - \frac{v^2}{c^2}}} = e \cdot v \cdot B \qquad \textbf{0.3}$$

In case of a mistake, partial points can be given for intermediate steps (up to max 0.2). Examples:

Example: Lorentz force **0.1**

Example: $\frac{\gamma \cdot m_p \cdot v^2}{r}$ **0.1**

Energy:

$$E = (\gamma - 1) \cdot m_p \cdot c^2 \simeq \gamma \cdot m_p \cdot c^2 \to \gamma = \frac{E}{m_p c^2}$$

Therefore:

$$\frac{E \cdot v}{c^2 \cdot r} = e \cdot B \qquad \textbf{0.3}$$

With

$$v \simeq c \quad \text{and} \quad r = \frac{L}{2\pi}$$

follows:

$$\to B = \frac{2\pi \cdot E}{e \cdot c \cdot L} \qquad \textbf{0.2}$$

Solution:

$$B = 5.50\text{T} \qquad \textbf{0.2}$$

Penalty for < 2 or > 4 significant digits **-0.1**

Calculation without approximations is also correct but does not give more points

$$B = \frac{2\pi \cdot m_p \cdot c}{e \cdot L} \cdot \sqrt{\left(\frac{E}{m_p \cdot c^2}\right)^2 - \left(1 + \frac{m \cdot c^2}{E}\right)^2} \qquad \textbf{0.5}$$

Penalty for each algebraic mistake **-0.1**

Classical calulation gives completely wrong result and maximum 0.3 pt **[0.3]**

$$\frac{m_p \cdot v^2}{r} = e \cdot v \cdot B \qquad \textbf{0.1}$$

$$B = \frac{2\pi}{L \cdot e}\sqrt{2 \cdot m_p \cdot E} \qquad \textbf{0.1}$$

$$B = 0.0901\mathrm{T}$$	**0.1**
Penalty for < 2 or > 4 significant digits	**-0.1**

A4 (1.0 pt) An accelerated charged particle radiates energy in the form of electromagnetic waves. The radiated power P_{rad} of a charged particle that circulates with a constant angular velocity depends only on its acceleration a, its charge q, the speed of light c and the permittivity of free space ϵ_0. Use a dimensional analysis to find an expression for the radiated power P_{rad}.

Solution A4: **[1.0]**

Ansatz:

$$P_{rad} = a^\alpha \cdot q^\beta \cdot c^\gamma \cdot \epsilon_0^\delta \qquad \mathbf{0.2}$$

Dimensions: [a]=ms^{-2}, [q]=C=As, [c]=ms^{-1},[ϵ_0]=As(Vm)$^{-1}$=A^{2}s^2(Nm^2)$^{-1}$=A^{2}s^4(kgm^3)$^{-1}$

All dimensions correct **0.3**

Three dimensions correct **0.2**

Two dimensions correct **0.1**

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if dimensions: N and Coulomb [ϵ_0]= C^2(Nm^2)$^{-1}$

$$\frac{\mathrm{m}^\alpha}{\mathrm{s}^{2\alpha}} \cdot \mathrm{C}^\beta \cdot \frac{\mathrm{m}^\gamma}{\mathrm{s}^\gamma} \cdot \frac{\mathrm{C}^{2\delta}}{\mathrm{N}^\delta \cdot \mathrm{m}^{2\delta}} = \frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}} \qquad \mathbf{0.1}$$

From this follows:

$$\mathrm{N} :\rightarrow \delta = -1, \quad \mathrm{C} :\rightarrow \beta + 2 \cdot \delta = 0, \quad \mathrm{m} :\rightarrow \alpha + \gamma - 2\delta = 1, \quad \mathrm{s} :\rightarrow 2 \cdot \alpha + \gamma = 1 \qquad \mathbf{0.2}$$

Two equations correct **0.1**

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1 \qquad \mathbf{0.1}$$

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if dimensions: N and As [ϵ_0]=A^{2}s^2(Nm^2)$^{-1}$

$$\frac{\mathrm{m}^\alpha}{\mathrm{s}^{2\alpha}} \cdot \mathrm{A}^\beta \cdot \mathrm{s}^\beta \cdot \frac{\mathrm{m}^\gamma}{\mathrm{s}^\gamma} \cdot \frac{\mathrm{A}^{2\delta} \cdot \mathrm{s}^{2\delta}}{\mathrm{N}^\delta \cdot \mathrm{m}^{2\delta}} = \frac{\mathrm{N} \cdot \mathrm{m}}{\mathrm{s}} \qquad \mathbf{0.1}$$

From this follows:

$$N :\rightarrow \delta = -1, \quad \mathrm{A} :\rightarrow \beta + 2 \cdot \delta = 0, \quad \mathrm{m} :\rightarrow \alpha + \gamma - 2\delta = 1, \quad \mathrm{s} :\rightarrow -2 \cdot \alpha + \beta - \gamma + 2\delta = -1 \qquad \mathbf{0.2}$$

Two equations correct **0.1**

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1 \qquad \mathbf{0.1}$$

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if dimensions: kg and As [ϵ_0]=A^{2}s^4(kg$\cdot$ m^3)$^{-1}$

$$\frac{\mathrm{m}^\alpha}{\mathrm{s}^{2\alpha}} \cdot \mathrm{A}^\beta \cdot \mathrm{s}^\beta \cdot \frac{\mathrm{m}^\gamma}{\mathrm{s}^\gamma} \cdot \frac{\mathrm{A}^{2\delta} \cdot \mathrm{s}^{4\delta}}{\mathrm{kg}^\delta \cdot \mathrm{m}^{3\delta}} = \frac{\mathrm{kg} \cdot m^2}{s^3} \qquad \mathbf{0.1}$$

From this follows:

$$\text{kg} :\rightarrow \delta = -1, \quad \text{A} :\rightarrow \beta + 2\cdot\delta = 0, \quad \text{m} :\rightarrow \alpha + \gamma - 3\delta = 2, \quad \text{s} :\rightarrow -2\cdot\alpha + \beta - \gamma + 4\delta = -3$$ **0.2**

Two equations correct **0.1**

And therefore:

$$\rightarrow \alpha = 2, \beta = 2, \gamma = -3, \delta = -1$$ **0.1**

Radiated Power:

$$P_{rad} \propto \frac{a^2 \cdot q^2}{c^3 \cdot \epsilon_0}$$ **0.1**

Other solutions with other units are possible and are accepted
No solution but realise that unit of charge must vanish $\beta = 2\delta$ **0.2**

A5 (1.0 pt) Calculate the total radiated power P_{tot} of the LHC for a proton energy of $E = 7.00$ TeV (Note table 1). You may use appropriate approximations.

Solution A5: **[1.0]**

Radiated Power:

$$P_{rad} = \frac{\gamma^4 \cdot a^2 \cdot e^2}{6\pi \cdot c^3 \cdot \epsilon_0}$$ **0.1**

Energy:

$$E = (\gamma - 1)m_p \cdot c^2 \quad \text{or equally valid} \quad E \simeq \gamma \cdot m_p \cdot c^2$$ **0.2**

Acceleration:

$$a \simeq \frac{c^2}{r} \quad \text{with} \quad r = \frac{L}{2\pi}$$ **0.2**

Therefore:

$$P_{rad} = (\frac{E}{m_p c^2} + 1)^4 \cdot \frac{e^2 \cdot c}{6\pi\epsilon_0 \cdot r^2} \quad \text{or} \quad (\frac{E}{m_p c^2})^4 \cdot \frac{e^2 \cdot c}{6\pi\epsilon_0 \cdot r^2}$$ **0.3**

(not required $P_{rad} = 7.94 \cdot 10^{-12}$W)

Total radiated power:

$$P_{tot} = 2 \cdot 2808 \cdot 1.15 \cdot 10^{11} \cdot P_{rad} = 5.13\text{kW}$$ **0.2**

penalty for missing factor 2 (for the two beams): **-0.1** **-0.1**
penalty for wrong numbers 2808 and/or $1.15 \cdot 10^{11}$ (numbers come from table 1): **-0.1** **-0.1**

A6 (1.5 pt) Determine the time T that the protons need to pass through this field.

Solution A6: **[1.5]**

2nd Newton's law

$$F = \frac{dp}{dt} \quad \text{leads to}$$ **0.2**

$$\frac{V \cdot e}{d} = \frac{p_f - p_i}{T} \text{ with } p_i = 0$$ **0.3**

Conservation of energy:

$$E_{tot} = m \cdot c^2 + e \cdot V$$ **0.2**

Since

$$E_{tot}^2 = (m \cdot c^2)^2 + (p_f \cdot c)^2$$ **0.2**

$$\rightarrow p_f = \frac{1}{c} \cdot \sqrt{(m \cdot c^2 + e \cdot V)^2 - (m \cdot c^2)^2} = \sqrt{2e \cdot m \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$ **0.2**

$$\rightarrow T = \frac{d \cdot p_f}{V \cdot e} = \frac{d}{V \cdot e}\sqrt{2e \cdot m_p \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$ **0.3**

$$T = 218\text{ns}$$ **0.1**

Alternative solution **[1.5]**

2nd Newton's law

$$F = \frac{dp}{dt} \quad \text{leads to}$$ **0.2**

$$\frac{V \cdot e}{d} = \frac{p_f - p_i}{T} \text{ with } p_i = 0$$ **0.3**

velocity from A1 or from conservation of energy

$$v = c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e}\right)^2}$$ **0.2**

and hence for γ

$$\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}} = 1 + \frac{e \cdot V}{m_p \cdot c^2}$$ **0.2**

$$\rightarrow p_f = \gamma \cdot m_p \cdot v = \left(1 + \frac{e \cdot V}{m_p \cdot c^2}\right) \cdot m_p \cdot c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + V \cdot e}\right)^2}$$ **0.2**

$$\rightarrow T = \frac{d \cdot p_f}{V \cdot e} = \frac{d \cdot m_p \cdot c}{V \cdot e} \cdot \sqrt{\left(\frac{m_p \cdot c^2 + e \cdot V}{m_p \cdot c^2}\right)^2 - 1} = \frac{d}{V \cdot e}\sqrt{2e \cdot m_p \cdot V + \left(\frac{e \cdot V}{c}\right)^2}$$ **0.3**

$$T = 218\text{ns}$$ **0.1**

Alternative solution: integrate time **[1.5]**

Energy increases linearly with distance x

$$E(x) = \frac{e \cdot V \cdot x}{d}$$ **0.2**

$$t = \int dt = \int_0^d \frac{dx}{v(x)}$$ **0.2**

$$v(x) = c \cdot \sqrt{1 - \left(\frac{m_p \cdot c^2}{m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d}}\right)^2} = c \cdot \frac{\sqrt{\left(m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d}\right)^2 - (m_p \cdot c^2)^2}}{m_p \cdot c^2 + \frac{e \cdot V \cdot x}{d}}$$

$$= c \cdot \frac{\sqrt{\left(1 + \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2}\right)^2 - 1}}{1 + \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2}}$$ **0.2**

$$\text{Substitution}: \xi = \frac{e \cdot V \cdot x}{d \cdot m_p \cdot c^2} \qquad \frac{d\xi}{dx} = \frac{e \cdot V}{d \cdot m_p \cdot c^2}$$ **0.2**

$$\rightarrow t = \frac{1}{c} \int_0^b \frac{1 + \xi}{\sqrt{(1+\xi)^2 - 1}} \frac{d \cdot m_p \cdot c^2}{e \cdot V} d\xi \qquad b = \frac{e \cdot V}{m_p \cdot c^2}$$ **0.2**

$$1 + \xi := \cosh(s) \qquad \frac{d\xi}{ds} = \sinh(s)$$ **0.1**

$$t = \frac{m_p \cdot c \cdot d}{e \cdot V} \int \frac{\cosh(s) \cdot \sinh(s) ds}{\sqrt{\cosh^2(s) - 1}} = \frac{m_p \cdot c \cdot d}{e \cdot V} [\sinh(s)]_{b_1}^{b_2}$$ **0.2**

$$\text{with} \quad b_1 = \cosh^{-1}(1), \quad b_2 = \cosh^{-1}\left(1 + \frac{e \cdot V}{m_p \cdot c^2}\right)$$ **0.1**

$$T = 218\text{ns}$$ **0.1**

Alternative: differential equation **[1.5]**

$$F = \frac{\mathrm{d}p}{\mathrm{d}t}$$ **0.2**

$$\rightarrow \frac{V \cdot e}{d} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{m \cdot a\left(1 - \frac{v^2}{c^2}\right) + m \cdot a \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \gamma^3 \cdot m \cdot a$$ **0.4**

$$a = \ddot{s} = \frac{V \cdot e}{d \cdot m}\left(1 - \frac{\dot{s}^2}{c^2}\right)^{\frac{3}{2}}$$ **0.3**

$$\text{Ansatz}: \; s(t) = \sqrt{i^2 \cdot t^2 + k} - l \; \text{ with boundary conditions } s(0) = 0, v(0) = 0$$ **0.1**

$$\rightarrow s(t) = \frac{c}{V \cdot e}\left(\sqrt{e^2 \cdot V^2 \cdot t^2 + c^2 \cdot m^2 \cdot d^2} - c \cdot m \cdot d\right)$$ **0.2**

$$s = d \rightarrow T = \frac{d}{V \cdot e}\sqrt{\left(\frac{V \cdot e}{c}\right)^2 + 2V \cdot e \cdot m}$$ **0.2**

$$T = 218\text{ns}$$ **0.1**

classical solution: **[0.4]**

$$F = \frac{V \cdot e}{d} \rightarrow \text{acceleration } a = \frac{F}{m_p} = \frac{V \cdot e}{m_p \cdot d}$$ **0.1**

$$d = \frac{1}{2} \cdot a \cdot T^2 \rightarrow T = \sqrt{\frac{2d}{a}}$$ **0.1**

And hence for the time

$$T = d \cdot \sqrt{\frac{2 \cdot m_p}{V \cdot e}}$$ **0.1**

$$T = 194\text{ns}$$ **0.1**

Part B. Particle identification (4 points)

B1 (0.8 pt) Express the particle rest mass m in terms of the momentum p, the flight length l and the flight time t assuming that the particles with elementary charge e travel with velocity close to c on straight tracks in the ToF detector and that it travels perpendicular to the two detection planes (see Figure 2).

Solution B1: **[0.8]**

with velocity

$$v = \frac{l}{t}$$ **0.1**

relativistic momentum

$$p = \frac{m \cdot v}{\sqrt{1 - \frac{v^2}{c^2}}}$$ **0.2**

gets

$$p = \frac{m \cdot l}{t \cdot \sqrt{1 - \frac{l^2}{t^2 \cdot c^2}}}$$ **0.2**

$\rightarrow$ mass

$$m = \frac{p \cdot t}{l} \cdot \sqrt{1 - \frac{l^2}{t^2 \cdot c^2}} = \frac{p}{l \cdot c} \cdot \sqrt{t^2 \cdot c^2 - l^2}$$ **0.3**

Alternative **[0.8]**

with flight distance: l, flight time t gets:

$$t = \frac{l}{(c \cdot \beta)}$$ **0.1**

relativistic momentum

$$p = \frac{m \cdot \beta \cdot c}{\sqrt{1 - \beta^2}}$$

therefore the velocity:

$$\beta = \frac{p}{\sqrt{m^2 \cdot c^2 + p^2}}$$ **0.2**

insert into the expression for t:

$$t = l \frac{\sqrt{m^2 \cdot c^2 + p^2}}{c \cdot p}$$ **0.2**

$\rightarrow$ mass:

$$m = \sqrt{\left(\frac{p \cdot t}{l}\right)^2 - \left(\frac{p}{c}\right)^2} = \frac{p}{l \cdot c} \sqrt{(t \cdot c)^2 - (l)^2}$$ **0.3**

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non-relativistic solution: **[0.0]**

flight time: $t = l/v$ velocity:

$$v = \frac{p}{m} \rightarrow t = \frac{l \cdot m}{p} \quad \text{and} \quad m = \frac{p \cdot t}{l}$$

this solution gives no points **0.0**

B2 (0.7 pt) Calculate the minimal length of a ToF detector that allows to safely distinguish a charged kaon from a charged pion given both their momenta are measured to be 1.00 GeV/c. For a good separation it is required that the difference in the time-of-flight is larger than three times the time resolution of the detector. The typical resolution of a ToF detector is 150 ps (1 ps = 10^{-12} s).

Solution B2: **[0.7]**

Flight time difference between kaon and pion

$$\Delta t = 450\text{ps} = 450 \cdot 10^{-12}\text{s} \qquad \textbf{0.1}$$

Flight time difference between kaon and pion

$$\Delta t = \frac{l}{cp}(\sqrt{m_\pi^2 \cdot c^2 + p^2} - \sqrt{m_K^2 \cdot c^2 + p^2}) = 450\text{ps} = 450 \cdot 10^{-12}\text{s} \qquad \textbf{0.2}$$

$$\rightarrow l = \frac{\Delta t \cdot p}{\sqrt{m_K^2 + p^2/c^2} - \sqrt{m_\pi^2 + p^2/c^2}} \qquad \textbf{0.2}$$

$$\sqrt{m_K^2 + p^2/c^2} = 1.115 \text{ GeV}/c^2 \text{and} \sqrt{m_\pi^2 + p^2/c^2} = 1.010 \text{ GeV}/c^2$$

$$l = 450 \cdot 10^{-12} \cdot \frac{1}{1.115 - 1.010} \text{s GeV}c^2/(\text{GeV}c) \qquad \textbf{0.1}$$

$$l = 4285.7 10^{-12}\text{s} \cdot c = 4285.7 \cdot 10^{-12} \cdot 2.998 \cdot 10^8\text{m} = 1.28\text{m} \qquad \textbf{0.1}$$

Penalty for < 2 or > 4 significant digits **-0.1**

Non-relativistic solution: **[0.3]**

Flight time difference between kaon and pion

$$\Delta t = \frac{l}{p}(m_K - m_\pi) = 450ps = 450 \cdot 10^{-12}\text{s} \qquad \textbf{0.1}$$

length:

$$l = \frac{\Delta tp}{m_K - m_\Pi} = \frac{450 \cdot 10^{-12}\text{s} \cdot 1\text{GeV}/c}{(0.498 - 0.135)\text{GeV}/c^2} \qquad \textbf{0.1}$$

$$l = 450 \cdot 10^{-12}/0.363 \cdot cs = 450 \cdot 10^{-12}/0.363 \cdot 2.998 \cdot 10^8\text{m}$$

$$l = 3716 \cdot 10^{-4}\text{m} = 0.372\text{m} \qquad \textbf{0.1}$$

Penalty for < 2 or > 4 significant digits **-0.1**

B3 (1.7 pt) Express the particle mass as a function of the magnetic flux density B, the radius R of the ToF tube, fundamental constants and the measured quantities: radius r of the track and time-of-flight t.

Solution B3: **[1.7]**

Particle is travelling perpendicular to the beam line hence the track length is given by the length of the arc
Lorentz force → transverse momentum, since there is no longitudinal momentum, the momentum is the same as the transverse momentum
Use formula from B1 to calculate the mass
track length: length of arc

$$l = 2 \cdot r \cdot \mathrm{asin} \frac{R}{2 \cdot r}$$ **0.5**

penalty for just taking a straight track ($l = R$) **-0.4**

partial points for intermediate steps, maximum 0.4
Lorentz force

$$\frac{\gamma \cdot m \cdot v_t^2}{r} = e \cdot v_t \cdot B \rightarrow p_T = r \cdot e \cdot B$$ **0.4**

partial points for intermediate steps, maximum 0.3
longitudinal momentum=0 → $p = p_T$ **0.1**

momentum

$$p = e \cdot r \cdot B$$ **0.1**

$$m = \sqrt{\left(\frac{p \cdot t}{l}\right)^2 - \left(\frac{p}{c}\right)^2} = e \cdot r \cdot B \cdot \sqrt{\left(\frac{t}{2r \cdot \mathrm{asin} \frac{R}{2r})}\right)^2 - \left(\frac{1}{c}\right)^2}$$ **0.6**

partial points for intermediate steps, maximum 0.5

Non-relativistic: track length: length of arc **[0.9]**

$$l = 2 \cdot r \cdot \mathrm{asin} \frac{R}{2 \cdot r}$$ **0.5**

penalty for just taking a straight track ($l = R$) **-0.4**

partial points for intermediate steps, maximum 0.4

$$m = \frac{p \cdot t}{l} = \frac{e \cdot r \cdot B \cdot t}{2r \cdot \mathrm{asin} \frac{R}{2r}} = \frac{e \cdot B \cdot t}{2 \cdot \mathrm{asin} \frac{R}{2r}}$$ **0.4**

partial points for intermediate steps, maximum 0.3

B4 (0.8 pt) Identify the four particles by calculating their mass.

Particle	Radius r [m]	Time of flight [ns]
A	5.10	20
B	2.94	14
C	6.06	18
D	2.32	25

Solution B4: **[0.8]**

Particle	arc [m]	p $[\frac{MeV}{c}]$	p $[\frac{mkg}{s}]$ 10^{-19}	pt/l $[\frac{MeVs}{cm}]$ 10^{-6}	pt/l $[\frac{MeV}{c^2}]$	pt/l [kg] 10^{-27}	Mass $[\frac{MeV}{c^2}]$	Mass [kg] 10^{-27}
A	3.786	764.47	4.0855	4.038	1210.6	2.158	938.65	1.673
B	4.002	440.69	2.3552	1.542	462.2	0.824	139.32	0.248
C	3.760	908.37	4.8546	4.349	1303.7	2.324	935.10	1.667
D	4.283	347.76	1.8585	2.030	608.6	1.085	499.44	0.890

Particles A and C are protons, B is a Pion and D a Kaon

correct mass and identification: per particle **0.2**

penalty for correct mass but no or wrong identification **for 1 or 2 particles** **-0.1**

penalty for correct mass but no or wrong identification **for 3 or 4 particles** **-0.2**

wrong mass, correct momentum:**per particle** **0.1**

wrong momentum, correct arc **for 3 or 4 particles** **0.2**

wrong momentum, correct arc **for 1 or 2 particles** **0.1**

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non relativistic solution $m = pt/l$ **Particle identification is not possible** **[0.4]**

Particle	arc [m]	p $[\frac{MeV}{c}]$	p $[\frac{mkg}{s}]$ 10^{-19}	$m = p \cdot t/l$ $[\frac{MeVs}{cm}]$ 10^{-6}	$m = p \cdot t/l$ $[\frac{MeV}{c^2}]$	$m = p \cdot t/l$ [kg] 10^{-27}
A	3.786	764.47	4.0855	4.038	1210.6	2.158
B	4.010	440.69	2.3552	1.542	462.2	0.824
C	3.760	908.37	4.8546	4.349	1303.7	2.324
D	4.283	347.76	1.8585	2.030	608.6	1.085

correct mass or correct momentum: **per particle** **0.1**

wrong momentum, correct arc **for 3 or 4 particles** **0.2**

wrong momentum, correct arc **for 1 or 2 particles** **0.1**